

Functions

Operations

- $(f + g)(x) = f(x) + g(x)$
- $(f - g)(x) = f(x) - g(x)$
- $(f \cdot g)(x) = f(x) \cdot g(x)$
- $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$
- $(f \circ g)(x) = f(g(x))$ - COMPOSITE FUNCTION

Domains: Consist of the x - values that are in the domains for *both* f and g . Also, the domain of the quotient function does not contain any x - value for which $g(x) = 0$.

Examples:

- Let $f(x) = 5x^3 + 1$ and $g(x) = x^2 - 4$. What are $f + g$ and $f - g$? What are the domains?

$$\begin{aligned} f(x) + g(x) &= (5x^3 + 1) + (x^2 - 4) = 5x^3 + 1 + x^2 - 4 = \boxed{5x^3 + x^2 - 3} \quad D: \{\mathbb{R}\} \\ f(x) - g(x) &= (5x^3 + 1) - (x^2 - 4) = 5x^3 + 1 - x^2 + 4 = \boxed{5x^3 - x^2 + 5} \end{aligned}$$

- Let $f(x) = x^2 + x - 6$ and $g(x) = x - 2$. What are $f \cdot g$ and $\frac{f}{g}$ and their domains?

$$\begin{aligned} f(x)g(x) &= (x^2 + x - 6)(x - 2) = x^3 - 2x^2 + x^2 - 2x - 6x + 12 = \boxed{x^3 - x^2 - 8x + 12} \\ \frac{f(x)}{g(x)} &= \frac{x^2 + x - 6}{x - 2} \quad D: \{\mathbb{R} \mid x \neq 2\} \end{aligned}$$

- Let $f(x) = x^2 + 1$ and $g(x) = x - 2$. What is $(g \circ f)(-2)$?

$$g(f(-2)) = g((-2)^2 + 1) = g(5) = 5 - 2 = \boxed{3}$$

- For same functions in previous example, find $(f \circ g)(x)$, $g(f(x))$, $(f \circ f)(3)$

$$\begin{aligned} f(g(x)) &= f(x - 2) = (x - 2)^2 + 1 = (x - 2)(x - 2) + 1 \\ &= x^2 - 4x + 4 + 1 \\ &= \boxed{x^2 - 4x + 5} \end{aligned}$$

$$\begin{aligned} g(f(x)) &= g(x^2 + 1) = (x^2 + 1) - 2 \\ &= \boxed{x^2 - 1} \end{aligned}$$

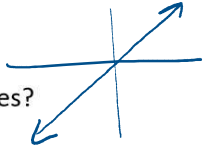
$$\begin{aligned} f(f(3)) &= f(3^2 + 1) = f(10) = (10)^2 + 1 \\ &= \boxed{101} \end{aligned}$$

$$\{(2,1), (3,2), (4,3), (2,2)\}$$

x	$f(x)$	x	$f^{-1}(x)$
0	1	1	0
2	3	3	2
4	5	5	4

Inverses

- If a relation pairs element a of its domain to element b of its range, the **inverse relation** pairs b with a . So, if (a, b) is an ordered pair of a relation, then (b, a) is an ordered pair of its inverse.
- The graphs of a relation and its inverse are *reflections* of each other in the line $y = x$.
- How to determine if an expression has an inverse? How to verify two functions are inverses?
 - A 1-1 function has an inverse that is also a function.
 - Horizontal Line Test (for every y there is exactly one x)
 - $f(g(x)) = x$ and $g(f(x)) = x$ if the two functions are inverses of each other.
 - If f and f^{-1} are inverse functions, then $(f^{-1} \circ f)(x) = x$ and $(f \circ f^{-1})(x) = x$ for x in the domain of f and f^{-1} , respectively.



$f^{-1}(x)$ "The inverse of f "

- To determine inverse of a function:
 - Replace $f(x)$ with y , if needed.
 - Switch x and y .
 - Solve for "new" y . This is your inverse function $f^{-1}(x)$. "f inverse" "inverse of f"

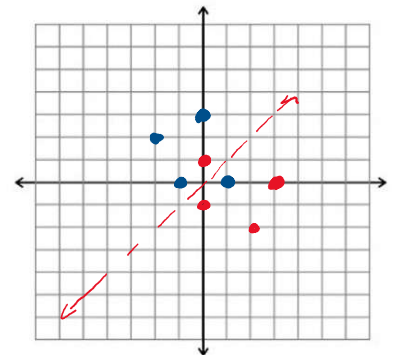
Examples

- Relation v

x	-2	-1	0	1
y	2	0	3	0

- What is the inverse of relation v ?
- What are the graphs of v and its inverse?

x	2	0	3	0
$f^{-1}(x)$	-2	-1	0	1



- What is the inverse of the relation described by $y = 5x^2 + 2$?

$$\begin{aligned}
 x &= 5y^2 + 2 \\
 x - 2 &= 5y^2 \\
 \frac{x-2}{5} &= y^2 \\
 \pm \sqrt{\frac{x-2}{5}} &= y \\
 \pm \sqrt{\frac{x-2}{5}} &= \cancel{y} \quad f^{-1}(x)
 \end{aligned}$$

- Consider the function $g(x) = -\frac{2}{3}x + 7$.
 - What is the g^{-1} , the inverse of g ?

$$\begin{aligned}
 &\cancel{g(x)} = -\frac{2}{3}x + 7 \\
 &y = -\frac{2}{3}x + 7 \\
 &x = -\frac{2}{3}y + 7 \\
 &-\frac{2}{3}(x-7) = \left(-\frac{2}{3}y\right) - \frac{2}{3} \\
 &-\frac{2}{3}x + \frac{14}{3} = -\frac{2}{3}y - \frac{2}{3}
 \end{aligned}$$

- For $h(x) = \frac{3}{2-x}$, what is each of the following?

- $h^{-1}(x)$
- $(h \circ h^{-1})(2)$
- $(h^{-1} \circ h)(2)$

$$\begin{aligned}
 &y = \frac{3}{2-x} \\
 &(2-y)x = \frac{3}{2-y} \cdot (2-y)
 \end{aligned}$$

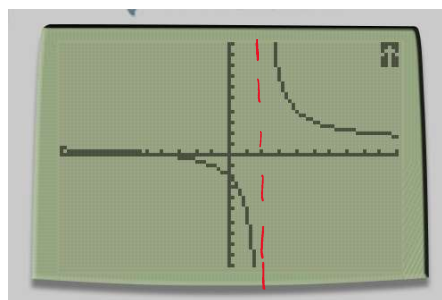
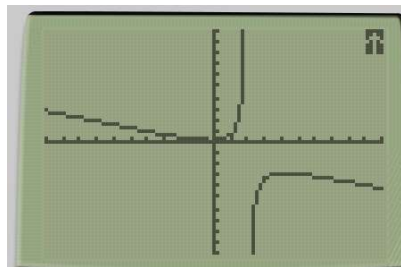
$$\begin{aligned}
 &x(2-y) = 3 \\
 &2x - xy = 3 \\
 &-xy = 3 - 2x \\
 &-x = \frac{3-2x}{-y}
 \end{aligned}$$

$$h^{-1}(x) = \frac{3-2x}{-x}$$

$$\begin{aligned}
 &h(h^{-1}(2)) \\
 &h\left(\frac{1}{2}\right) = \frac{3}{2-\frac{1}{2}}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{3}{\frac{3}{2}} = \frac{6}{3} = 2 \\
 &= 3 \div \frac{3}{2} = \frac{3}{1} \cdot \frac{2}{3} = \frac{6}{3} = 2
 \end{aligned}$$

$$\begin{aligned}
 &h^{-1}(h(2)) \\
 &\text{Undefined}
 \end{aligned}$$



$$x = 2$$