

Honors Algebra II

Function Operations, Rational Functions/Equations, and Asymptotes

Review

If $f(x) = 3x + 2$ and $g(x) = x^2 - 2x + 1$, find each of the following:

1. $f(x) + g(x) = (3x+2) + (x^2-2x+1) = \boxed{x^2 + x + 3}$
2. $f(x) - g(x) = (3x+2) - (x^2-2x+1) = 3x+2-x^2+2x-1 = \boxed{-x^2 + 5x + 1}$
3. $f(x) \cdot g(x) = (3x+2)(x^2-2x+1) = 3x^3 - 6x^2 + 3x + 2x^2 - 4x + 2 = \boxed{3x^3 - 4x^2 - x + 2}$
4. $f(g(x)) = f(x^2-2x+1) = 3(x^2-2x+1)+2 = 3x^2-6x+3+2 = \boxed{3x^2-6x+5}$
5. $g(f(x)) = g(3x+2) = (3x+2)^2 - 2(3x+2) + 1 = 9x^2 + 12x + 4 - 6x - 4 + 1 = \boxed{9x^2 + 6x + 1}$
6. $f(g(-5)) = f(36) = 3(36) + 2 = \boxed{110}$
7. $f(-3) - g(2) = [3(-3)+2] - [2^2-2(2)+1] = -7-1 = \boxed{-8}$
8. Determine the inverse of $f(x) = \frac{x-8}{3}$ $x = \frac{y-8}{3}$ $3x = y-8$ $\boxed{3x+8 = y = f^{-1}(x)}$

Simplify each expression.

9. $\frac{x^2-1}{x^2-4} \cdot \frac{x^2-5x-14}{x^2-6x-7} = \frac{(x-1)\cancel{(x+1)}}{(x-2)\cancel{(x+2)}} \cdot \frac{\cancel{(x-7)}(x+2)}{\cancel{(x-7)}(x+1)} = \frac{x-1}{x-2} \quad x \neq 2, -2, -1$
10. $\frac{-16xy}{27z} \div \frac{8x^2}{15z^3} = \frac{-16xy}{27z} \cdot \frac{15z^3}{8x^2} = \frac{-10yz^2}{9x} \quad z \neq 0$
11. $\frac{\frac{x^2+3x-18}{x+4}}{\frac{x^2+7x+6}{x+4}} = \frac{(x+6)\cancel{(x-3)}}{(x+4)} \cdot \frac{(x+4)}{(x+6)(x+1)} = \frac{x-3}{x+1} \quad x \neq -1, -4, -6$
12. $\frac{y}{2x} + \frac{4y}{3x^2} - \frac{5}{6xy^2} = \frac{y(3xy^2) + 4y(2y^2) - 5(x)}{6x^2y^2} = \frac{3xy^3 + 8y^3 - 5x}{6x^2y^2} \quad x \neq 0, y \neq 0$
13. $\frac{2}{x^2-3x-10} - \frac{6}{x^2-8x+15} = \frac{2(x-5)-6(x+2)}{(x-5)(x+2)(x-3)} = \frac{2x-6-6x-12}{(x-5)(x+2)(x-3)} = \frac{-4x-18}{(x-5)(x+2)(x-3)} \quad x \neq 5, -2, 3$
14. $\frac{\frac{3}{2x} + \frac{x}{5}}{\frac{2x+3}{x+1} + \frac{x+1}{2x+3}} = \frac{\frac{3(2x+3) + x(2x+3)}{2x(2x+3)}}{\frac{2(2x+3) + 5(x+1)}{(x+1)(2x+3)}} = \frac{3x+3-2x^2-3x}{4x^2+11x+5} = \frac{-2x^2+3}{4x^2+11x+5} \quad x \neq -\frac{1}{2}, -1$

Solve the following:

15. $\frac{1}{3} + \frac{4}{x-2} = 6$ $3(x-2) \left(\frac{1}{3} \right) + 3(x-2) \left(\frac{4}{x-2} \right) = 3(x-2)(6)$ $x-2+12 = 18x-36$
 $-x+36 = -x+36$
 $\frac{46}{17} = \frac{17x}{17}$ $x = \frac{46}{17}$
16. $\frac{3}{x+2} + \frac{1}{x} = 0$ $x(x+2) \left(\frac{3}{x+2} \right) + x(x+2) \left(\frac{1}{x} \right) = x(x+2)(0)$ $3x+x+2=0$
 $4x+2=0$
 $-2 = -4x$
 $\frac{-2}{-4} = \frac{-x}{-4}$
 $\frac{1}{2} = x$ $x = \frac{1}{2}$

$$LCD: (2x-3)(x+1)$$

$$17. \frac{4}{2x-3} + \frac{x}{x+1} = \frac{-8x}{2x^2-x-3}$$

$$(2x-3)(x+1) \left(\frac{4}{2x-3} + \frac{x}{x+1} \right) = (2x-3)(x+1) \left(\frac{-8x}{(2x-3)(x+1)} \right)$$

$$4(x+1) + x(2x-3) = -8x$$

$$4x+4 + 2x^2-3x = -8x$$

$$2x^2 + 9x + 4 = 0$$

$$(2x+1)(x+4) = 0$$

$$x = -\frac{1}{2}, -4$$

State all discontinuities and label each as removable or non-removable. State all vertical and/or horizontal asymptotes (Remember-asymptotes are equations; ie, $x = \underline{\hspace{1cm}}$ or $y = \underline{\hspace{1cm}}$).

$$18. f(x) = \frac{x+2}{x^2+6x+8} = \frac{(x+2)}{(x+4)(x+2)}$$

$$x = -2 \text{ (Removable)}$$

$$x = -4 \text{ (Non-Removable)}$$

$$\text{Horizontal Asymptote: } y = 0$$

$$\text{Vertical Asymptote: } x = -4$$

$$19. f(x) = \frac{x^2-9}{x^2-5x-24} = \frac{(x-3)(x+3)}{(x-8)(x+3)}$$

$$x = -3 \text{ (Removable)}$$

$$x = 8 \text{ (Non-Removable)}$$

$$\text{Horizontal Asymptote: } y = \frac{1}{1} = 1$$

$$\text{Vertical Asymptote: } x = 8$$

$$20. f(x) = \frac{x^2+4x+4}{x+2} = \frac{(x+2)(x+2)}{x+2}$$

$$x = -2 \text{ (Removable)}$$

$$\text{Horizontal/Vertical Asymptotes: None}$$