

Rational Functions

Format: $y = \frac{P(x)}{Q(x)}$

- Domain: Set of all reals *except* those which make the denominator zero!!!!
- If there is no value(s) for which the denominator is zero, then the function is said to be continuous

$$y = \frac{x}{x^2+8}$$
- If there is a value(s) where the denominator is zero, there is *discontinuity*.
 - *Removable Discontinuity*: Could you make the function continuous by redefining discontinuity point(s).

$$y = \frac{(x+3)(x+2)}{(x+2)}$$

What if you made $f(-2) = 1$? IE, do the y values get infinitely close to a value on both sides of the discontinuity point?

--Occur when you factor both the numerator and the denominator, and they share a factor (the discontinuity caused by this factor is removable)

- *Non-Removable Discontinuity*: If there is no value that you could replace in for the discontinuity point to make continuous.

$$y = \frac{x+4}{x-2}$$

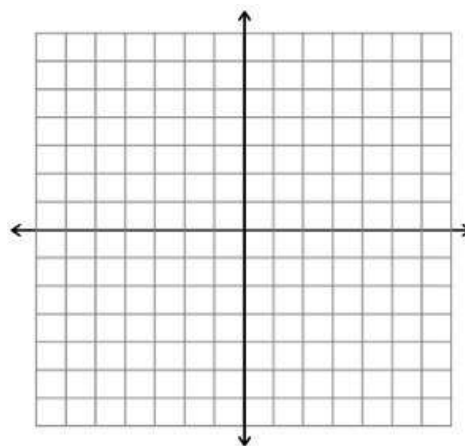
- *Asymptotes*:
 - *Vertical*: Occur when denominator of rational function is zero. Vertical asymptotes occur at the *line* $x = \underline{\hspace{1cm}}$
 - *Horizontal*:
 - If power of numerator greater than power of denominator: No horizontal asymptote.
 - If power of denominator greater than power of numerator: Horizontal asymptote at *line* $y = \underline{\hspace{1cm}}$
 - If the powers are the same, horizontal asymptote at *line* $y = \frac{a}{b}$ where a and b are the leading coefficients of the numerator and denominator.

Examples:

1. What are the domain and points of discontinuity of $y = \frac{x^2+4x+4}{x+2}$? Are the points of discontinuity removable or non-removable? What are the x and y intercepts?

2. What are the vertical asymptotes of the graph $y = \frac{x-3}{(x^2-3x+2)(x^2-7x+1)}$?

3. What is the horizontal asymptote of the graph of $y = \frac{x^2+1}{-3x+6}$? Graph the function.



Rational Expressions & Equations

Rational Expression: Quotient of two polynomials.

- *Simplest Form:* When both numerator and denominator are polynomials that no longer share any common divisors/factors.
- *Domain:* Comes from non-simplified form
- *Adding/Subtracting:* Find LCM of denominator.
 - Factor expressions completely.
 - LCM of the denominator (LCD) is product of all prime factors (unique) to their highest power

Examples:

1. What is $\frac{9x^2+6x}{36x+2}$ in simplest form? State any restrictions on the variable.
2. What is the product $\frac{x^2-3x+2}{x+2} \cdot \frac{x^2-36}{x^2+5x-6}$ in simplest form. State any restrictions on the variable.
3. What is the quotient $\frac{6x-3x^2}{36-x^2} \div \frac{x^3-x^2-2x}{x^2-5x-6}$ in simplest form? State any restrictions on the variable.
4. What is the LCM of $x^2 + 4x - 12$ and $x^2 - 6x + 8$?
5. What is the sum $\frac{4}{x^2+3x} + \frac{x-2}{x^2+6x+9}$ in simplest form? State any restrictions on the variable.

6. What is the difference $\frac{x+1}{x^2+2x-8} - \frac{x}{4x-8}$ in simplest form? State any restrictions on the variable.

7. What is a simpler form of $\frac{3x - \frac{1}{y}}{\frac{y^2}{x} + x}$

Solving Rational Equations:

- Determine LCD.
- Multiple ALL terms by LCD.
- Simplify (you shouldn't have any denominators left).
- Solve.
- Check for extraneous solutions (often occur if one of your solutions is a value which your original equation is undefined).

Examples:

1. What are the solutions of the rational equation?

a. $\frac{4}{x+2} = \frac{5}{2x+3}$

b. $\frac{1}{x^2-5x} + \frac{x-7}{x} = \frac{4}{x^2-5x}$

c. $\frac{5}{x+1} + \frac{x}{x-1} = 5$