

Solving Polynomials of Higher Degree

- Calculator Investigation: What degree polynomial regression equation would fit this data the best?

- Enter data into calculator. Stat -> Edit... -> L1, L2
- StatPlot -> Enter (Plot 1) -> On
 - o First plot selected, XList L1, YList L2
- Y = Be sure all are empty
- ZoomStat (view data set)
- Stat -> Calc -> LinReg(ax+b) L1, L2, Y1
- Stat -> Calc -> QuadReg L1, L2, Y2
- Stat -> Calc -> CubicReg L1, L2, Y3
- Graph. Which one is best fit for data?

x	f(x)
0	-6
1	-10
2	-14
3	-12
4	2
5	34

$$y = x^3 - 3x^2 - 2x - 6$$

- **Fundamental Theorem of Algebra:** If $P(x)$ is polynomial of degree $n \geq 1$, then $P(x) = 0$ has exactly n roots/solutions/zeros, including multiple and complex roots.

$$y = x^2 + 5x + 6$$

$$y = (x+3)(x+2)$$

$$y = (x-3)(x-2)$$

$$x+3=0 \quad x+2=0$$

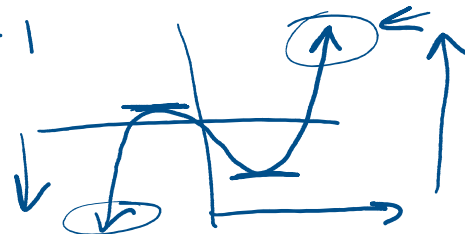
$$x=-3 \quad x=-2$$

- When working with polynomials:

- o $x^i - b$ is a linear factor of $P(x)$
- o b is labeled as **ZERO**, **ROOT** (**SOLUTION**), and **X-INTERCEPT**

- If the graph of a polynomial function has several turning points, the function can have a **relative maximum** and a **relative minimum**.

$$y = 1x^3 + 2x - 1$$



- Factor to get zeros/roots (solutions).

- o SEE WHAT YOU CAN PULL OUT! (Common factor--GCF!)

- o Sum or Difference of **perfect cubes**

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

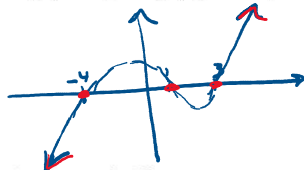
- o **Grouping**

$$ax + ay + bx + by = a(x+y) + b(x+y) = (x+y)(a+b)$$

- What are the zeros of $y = (x-3)(x+4)(x-1)$?

$$x-3=0 \quad x+4=0 \quad x-1=0$$

$$x=3 \quad x=-4 \quad x=1$$



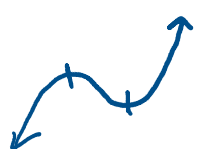
- What is the cubic function in standard form with zeros 1, -1, and 4?

$$y = (x-1)(x-1)(x-4) \quad y = (x^2 - 1)(x-4)$$

$$y = (x-1)(x+1)(x-4) \quad y = x^3 - 4x^2 - 1x + 4$$

- **Real Zeros of Polynomial Functions**

- o At most n real zeros
- o At most $n - 1$ turning points
- o $x = a$ is a **zero** of the function
- o $x = a$ is a **solution** of equation $f(x) = 0$



- $(x - a)$ is a factor of the polynomial
- $(a, 0)$ is an x-intercept of f
- Descartes' Rule of Signs
 - Positive real zeros: $P(x)$; Same as number of changes in sign of the coefficients or less by an even number.
 - Negative real zeros: $P(-x)$; Same as number of changes in sign of the coefficients or less by an even number.

- Find all real zeros of $f(x) = x^3 - 12x^2 + 36x$. Then determine the maximum number of turning points of the graph of the function.

Positive: 2, 0

$$\text{Negative: } f(-x) = (-x)^3 - 12(-x)^2 + 36(-x) \\ = -x^3 - 12x^2 - 36x$$

max turning points: 2

$$f(x) = x(x^2 - 12x + 36) \\ f(x) = x(x - 6)(x - 6) \\ x = 0 \quad x - 6 = 0 \quad x - 6 = 0 \\ x = 0 \quad x = 6 \quad x = 6$$

- What are the real solutions of the equation $2x^3 = -54$?

$$x^3 = \frac{-54}{2}$$

$$\sqrt[3]{x^3} = \sqrt[3]{-27}$$

$$x = -3$$

$$(-3)(-3)(-3) = -27$$

- What are the real solutions of $2x^3 + 5 = 3x^2 - 2x$? Use calculator.

- Sketch the graph of $f(x) = 2x^3 - 6x^2$

Step 1: Leading coefficient test Down to left, Up to Right

Step 2: Find the real zeros/x-intercepts

Step 3: Plot a few additional points

Step 4: Sketch

$$2x^3 - 6x^2 = 0 \\ 2x^2(x - 3) = 0$$

$$\frac{2x^2}{2} = \frac{0}{2}$$

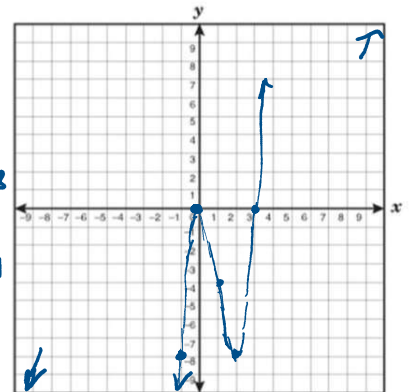
$$x^2 = 0$$

$$x = 0$$

$$x - 3 = 0 \\ x = 3$$

Double zero

x	y
-1	-2-8
1	-4
2	16-24 = -8
4	32



- Factor completely: $8x^3 = 27y^3$

$$8x^3 - 27y^3 = 0$$

$$(2x - 3y)(4x^2 + 6xy + 9y^2) = 0$$

$$(2x - 3y)(4x^2 + 6xy + 9y^2)$$

PRIME

$$x^2 \cdot x^2 \cdot x^2 = x^6$$

- Factor completely: $4x^6 + 256z^3 = 0$

$$4(\underline{x^6} + \underline{64z^3}) = 0$$

$$4(x^2 + 4z)(\underline{x^4 - 4x^2z + 16z^2}) = 0$$

- Find ALL solutions: $2x^3 = -54$ (See notes above)

$$2x^3 + 54 = 0$$

$$\frac{2(x^3 + 27)}{2} = \frac{0}{2}$$

$$\underline{x^3 + 27} = 0$$

$$(x+3)(x^2-3x+9)=0$$

$$x+3=0 \quad a=1 \quad b=-3 \quad c=9$$

$$\boxed{x = -3} \quad x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(9)}}{2(1)}$$

$$x = \frac{3 \pm \sqrt{-27}}{2}$$

$$\left\{ \begin{aligned} x &= \frac{3 \pm i\sqrt{27}}{2} = \frac{3 \pm i\sqrt{9 \cdot 3}}{2} \\ x &= \frac{3 \pm 3i\sqrt{3}}{2} \end{aligned} \right.$$

$$\boxed{x = \frac{3 \pm 3i\sqrt{3}}{2}}$$

- What is the linear model that best fits the data below? Use the model to estimate the egg consumption in 1995.

$$y = -2x + 4231.5$$

Year	US Per Capita Egg Consumption
1970	302
1980	266
1990	231
2000	247

$$\begin{aligned} Q \ 1995 \\ \approx 241.5 \end{aligned}$$

- What is the quadratic model that best fits the data above? Use the model to estimate the egg consumption in 1995.

$$y = .13x^2 - 518.1x + 518444.5$$

$$@ 1995 \approx 238.25$$

- Which is the better model for estimating egg consumption between 1970 and 2000?

Quadratic Model.

Dividing Polynomials

Long Division: Another way of factor a polynomial. Similar to numerical long division.

- Both the polynomial, $P(x)$ and divisor, $D(x)$ need to be in standard form and input zero coefficients when needed. Process stops when the degree of the remainder, $R(x)$ is less than the degree of the divisor.
- If no remainder is found, then the divisor is a factor of the polynomial.
- Sometimes beneficial to determine whether $x - a$ is a factor of the polynomial first. How do we do this?
 - If $x - a$ is a factor, then it is a zero of the polynomial (meaning... when you input a into the polynomial, it should output zero).

Synthetic Division: Yet another way of factoring a polynomial.

- Write down just the coefficients of the polynomial, in standard form (don't include variables or exponents).
- Don't forget "place-holders" if needed
- Divisor—reverse the sign of $x - a$ term.

Remainder Theorem: If you divide $P(x)$ of degree $n \geq 1$ by $x - a$, then the remainder is equal to $P(a)$.

Examples:

- Use long division to divide $5x^2 + 2x + 3$ by $x + 1$. What are the quotient and remainder?

$$\begin{array}{r}
 \quad 5x - 3 \\
 x+1 \overline{) 5x^2 + 2x + 3} \\
 \underline{-5x^2 + 5x} \\
 -3x + 3 \\
 \underline{-(-3x - 3)} \\
 6
 \end{array}$$

$$\begin{array}{l}
 \text{Quotient: } x+1 \\
 5(-1)^2 + 2(-1) + 3 = 6 \\
 5 - 2 + 3
 \end{array}$$

- Is $x^2 - 2$ a factor of $P(x) = x^4 - x^2 - 2$? If it is, write $P(x)$ as a product of two factors.

$$\begin{array}{r}
 \quad x^2 + 1 \\
 x^2 - 2 \overline{) x^4 + 0x^3 - x^2 + 0x - 2} \\
 \underline{-x^4 + 2x^2} \\
 2x^2 + 0x - 2 \\
 \underline{-2x^2 + 2} \\
 0
 \end{array}$$

Yes!

$$\begin{array}{l}
 (x^2 + 1)(x^2 - 2) = \\
 x^4 - x^2 - 2
 \end{array}$$

- Use synthetic division to divide $4x^3 - 3x^2 + 2x - 3$ by $x - 1$. What are the quotient and the remainder?

$$\begin{array}{r|rrrr} 1 & 4 & -3 & 2 & -3 \\ & \downarrow & & & \\ & 4 & 1 & 3 & \\ \hline & 4 & 1 & 3 & \boxed{0} \\ & x^2 & x & \# & \end{array}$$

$$(x-1)(4x^2+1x+3)$$

- The polynomial $x^3 + 9x^2 + 23x + 15$ expresses the volume, in cubic inches, of a box, and the length is $(x + 5)$ inches. What are the other two dimensions of the box?



$$\begin{array}{r|rrrr} -5 & 1 & 9 & 23 & 15 \\ & \downarrow & & & \\ & -5 & -20 & -15 & \\ \hline & 1 & 4 & 3 & \boxed{0} \end{array}$$

Length: $x+5$

Height $\times$ width: $x^2 + 4x + 3$

$$(x+3)(x+1)$$

H $\times$ W

- Given that $P(x) = x^4 + 6x^3 + 9x^2 + 3x - 3$, what is $P(4)$?

$$\begin{array}{r|rrrrr} 4 & 1 & 6 & 9 & 3 & -3 \\ & \downarrow & & & & \\ & 4 & 40 & 196 & 796 & \\ \hline & 1 & 10 & 49 & 199 & \boxed{793} \end{array}$$

$$P(4) = 793$$