

Solving Polynomials of Higher Degree

- Calculator Investigation: What degree polynomial regression equation would fit this data the best?

- Enter data into calculator. Stat -> Edit... -> L1, L2
- StatPlot -> Enter (Plot 1) -> On
 - o First plot selected, XList L1, YList L2
- Y = Be sure all are empty
- ZoomStat (view data set)
- Stat -> Calc -> LinReg(ax+b) L1, L2, Y1
- Stat -> Calc -> QuadReg L1, L2, Y2
- Stat -> Calc -> CubicReg L1, L2, Y3
- Graph. Which one is best fit for data?

x	$f(x)$
0	-6
1	-10
2	-14
3	-12
4	2
5	34

- Fundamental Theorem of Algebra:** If $P(x)$ is polynomial of degree $n \geq 1$, then $P(x) = 0$ has exactly n roots/solutions/zeros, including multiple and complex roots.
- When working with polynomials:
 - o $x - b$ is a linear factor of $P(x)$
 - o b is labeled as ZERO, ROOT (SOLUTION), and X-INTERCEPT
- If the graph of a polynomial function has several turning points, the function can have a **relative maximum** and a **relative minimum**.
- Factor to get zeros/roots (solutions).
 - o SEE WHAT YOU CAN PULL OUT! (Common factor--GCF!)
 - o Sum or Difference of perfect cubes
 - $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
 - $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
 - o Grouping
 - $ax + ay + bx + by = a(x + y) + b(x + y) = (x + y)(a + b)$
- What are the zeros of $y = (x - 3)(x + 4)(x - 1)$?
- What is the cubic function in standard form with zeros 1, -1, and 4?
- Real Zeros of Polynomial Functions**
 - o At most n real zeros
 - o At most $n - 1$ turning points
 - o $x = a$ is a *zero* of the function
 - o $x = a$ is a *solution* of equation $f(x) = 0$

- $(x - a)$ is a *factor* of the polynomial
- $(a, 0)$ is an *x-intercept* of f
- Descartes' Rule of Signs
 - Positive real zero's: $P(x)$; Same as number of changes in sign of the coefficients or less by an even number.
 - Negative real zero's: $P(-x)$; Same as number of changes in sign of the coefficients or less by an even number.
- Find all real zeros of $f(x) = x^3 - 12x^2 + 36x$. Then determine the maximum number of turning points of the graph of the function.

- What are the *real* solutions of the equation $2x^3 = -54$?

- What are the real solutions of $2x^3 + 5 = 3x^2 - 2x$? Use calculator.

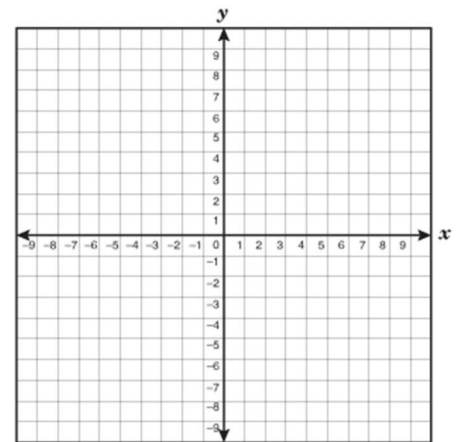
- Sketch the graph of $f(x) = 2x^3 - 6x^2$

Step 1: Leading coefficient test

Step 2: Find the real zeros/x-intercepts

Step 3: Plot a few additional points

Step 4: Sketch



- Factor *completely*: $8x^3 = 27y^3$

- Factor *completely*: $4x^6 + 256z^3 = 0$

- Find *ALL* solutions: $2x^3 = -54$ (See notes above)

- What is the linear model that best fits the data below? Use the model to estimate the egg consumption in 1995.

Year	US Per Capita Egg Consumption
1970	302
1980	266
1990	231
2000	247

- What is the quadratic model that best fits the data above? Use the model to estimate the egg consumption in 1995.
- Which is the better model for estimating egg consumption between 1970 and 2000?

Dividing Polynomials

Long Division: Another way of factor a polynomial. Similar to numerical long division.

- Both the polynomial, $P(x)$ and divisor, $D(x)$ need to be in standard form and input zero coefficients when needed. Process stops when the degree of the remainder, $R(x)$ is less than the degree of the divisor.
- If no remainder is found, then the divisor is a factor of the polynomial.
- Sometimes beneficial to determine whether $x - a$ is a factor of the polynomial first. How do we do this?
 - If $x - a$ is a factor, then it is a *zero* of the polynomial (meaning....when you input a into the polynomial, it should output zero).

Synthetic Division: Yet another way of factoring a polynomial.

- Write down just the coefficients of the polynomial, in standard form (don't include variables or exponents).
- Don't forget "place-holders" if needed
- Divisor—reverse the sign of $x - a$ term.

Remainder Theorem: If you divide $P(x)$ of degree $n \geq 1$ by $x - a$, then the remainder is equal to $P(a)$.

Examples:

- Use long division to divide $5x^2 + 2x + 3$ by $x + 1$. What are the quotient and remainder?

- Is $x^2 - 2$ a factor of $P(x) = x^4 - x^2 - 2$? If it is, write $P(x)$ as a product of two factors.

- Use synthetic division to divide $4x^3 - 3x^2 + 2x - 3$ by $x - 1$. What are the quotient and the remainder?
- The polynomial $x^3 + 9x^2 + 23x + 15$ expresses the volume, in cubic inches, of a box, and the length is $(x + 5)$ inches. What are the other two dimensions of the box?
- Given that $P(x) = x^4 + 6x^3 + 9x^2 + 3x - 3$, what is $P(4)$?

Theorems and Applications for Roots of Polynomials

Rational Root Theorem: Given $P(x) = a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$, where each coefficient is an integer. Two different types of roots possible.

- All of the integer roots must be factors of a_0 .
- All rational roots (fractions) must have reduced form $\frac{p}{q}$ where p is integer factor of a_0 and q is integer factor of a_n .
- Note: Once you determine one root, you can use long division or synthetic division to reduce polynomial to quadratic level, and then you can use quadratic formula, standard factoring, etc. to factor completely.

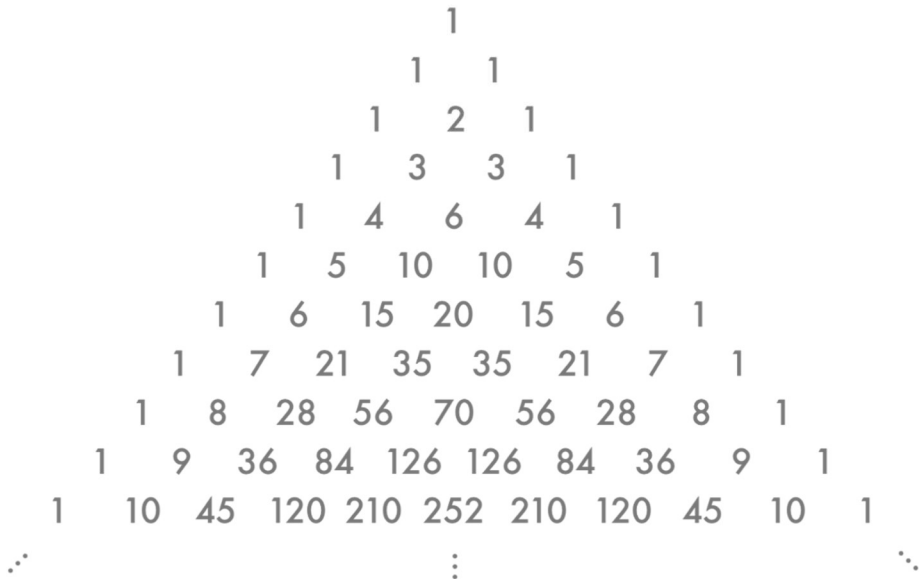
Note: Don't forget Descartes'

Examples

- What are the rational roots of $x^3 - 5x^2 - 2x + 10 = 0$?
- What are the rational roots of $x^3 - 2x^2 - 5x + 6 = 0$? Rewrite polynomial in factored format.
- What are the zeros of $f(x) = x^4 + 2x^3 - 4x^2 - 7x - 2$?

Binomial Theorem and Pascal's Triangle: Used to expand powers of binomials.

$$\begin{aligned}(a+b)^0 &= 1 \\(a+b)^1 &= a+b \\(a+b)^2 &= a^2+2ab+b^2 \\(a+b)^3 &= a^3+3a^2b+3ab^2+b^3 \\(a+b)^4 &= a^4+4a^3b+6a^2b^2+4ab^3+b^4 \\(a+b)^5 &= a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5\end{aligned}$$



- Binomial Theorem: $(a + b)^n = P_0a^n + P_1a^{n-1}b + P_2a^{n-2}b^2 + \dots + P_{n-1}ab^{n-1} + P_nb^n$

Examples

- What is the expansion of $(x + 3)^4$?

- What is the expansion of $(3 - z)^3$?