

Other Types of Equations

To Solve Radical Equations:

- Isolate the radical on one side of the equation
- Raise each side to the power suggested by the index
- Solve
 - CHECK for *extraneous* solutions!!!!

To Solve Radical Equation Involving 2 Radical Expressions or Exponents:

- Isolate one of the radical terms (Pick the more difficult term)
- Eliminate that term
- Simplify
- Eliminate 2nd radical term
 - CHECK for *extraneous* solutions!!!

To Solve Absolute Value

- Isolate the absolute value term
- Set up 2 equations
 - One as it is written
 - One as the negative of what is written

Examples:

- What is the solution of $\sqrt{x+4} + 6 = 7$?
 $\quad \quad \quad -6 \quad -6$

$$\begin{aligned} (\sqrt{x+4})^2 &= (1)^2 \\ x+4 &= 1 \\ -4 \quad -4 & \\ \hline x &= -3 \end{aligned} \quad \boxed{x = -3} \quad \checkmark$$

- What is the solution of $(6x+9)^{\frac{1}{3}} - 5 = -2$?
 $\quad \quad \quad +5 \quad +5$

$$\begin{aligned} (6x+9)^{\frac{1}{3}} &= 3 \\ (6x+9)^{\frac{1}{3}}^3 &= 3^3 \\ 6x+9 &= 27 \\ -9 \quad -9 & \\ \hline 6x &= 18 \\ \frac{6x}{6} &= \frac{18}{6} \\ x &= 3 \end{aligned} \quad \boxed{x = 3}$$

- A parabolic goblet with a cup that is as wide as it is tall holds $1.74r^3$ oz. of water when it is full, where r is the radius in inches of the circular rim. What is the radius, to the nearest hundredth of an inch, of a goblet that holds 9 oz?

$$\begin{aligned} 1.74r^3 &= 9 \\ \frac{1.74r^3}{1.74} &= \frac{9}{1.74} \\ r^3 &= 5.172 \\ \sqrt[3]{r^3} &= \sqrt[3]{5.172} \\ r &= 1.73'' \end{aligned}$$

- What is the solution of $\sqrt{5x+14} = x$?

$$\begin{aligned} (\sqrt{5x+14})^2 &= (x)^2 \\ 5x+14 &= x^2 \\ -5x \quad -14 \quad -5x \quad -14 & \\ \hline 0 &= x^2 - 5x - 14 \\ 0 &= (x+2)(x-7) \end{aligned} \quad \begin{aligned} \sqrt{5(7)+14} &\stackrel{?}{=} 7 \quad \checkmark \\ \sqrt{5(-2)+14} &\stackrel{?}{=} -2 \quad \times \\ \hline \boxed{x=7} \end{aligned}$$

$$x = 7, \quad \cancel{x = -2}$$

$$\begin{aligned} a^{\frac{m}{n}} &= \sqrt[n]{a^m} \\ [(x+1)^{\frac{4}{3}}]^{\frac{3}{4}} &= [1]^{\frac{4}{3}} = (\sqrt[n]{a})^m \\ 4^{\frac{3}{2}} &= (\sqrt{4})^3 = \frac{(x^2)^3}{x^2 x^2 x^2} \quad (3^{\frac{1}{3}})^3 \end{aligned}$$

- What is the solution of $2 + \sqrt{x-6} = \sqrt{x+10}$?

$$\begin{aligned}
 (2 + \sqrt{x-6})^2 &= (\sqrt{x+10})^2 \\
 (2 + \sqrt{x-6})(2 + \sqrt{x-6}) &= x+10 \\
 4 + 2\sqrt{x-6} + 2\sqrt{x-6} + (x-6) &= x+10 \\
 \cancel{4} + \cancel{2\sqrt{x-6}} + \cancel{2\sqrt{x-6}} + \cancel{x} - \cancel{6} &= \cancel{x} + 10 \\
 -2 + 4\sqrt{x-6} &= 10 \\
 4\sqrt{x-6} &= 12 \\
 \sqrt{x-6} &= 3 \\
 x-6 &= 9 \\
 x &= 15
 \end{aligned}$$

- Solve: $|x^2 + 6x| = 3x + 18$

$$\begin{aligned}
 x^2 + 6x &= 3x + 18 \\
 x^2 + 3x - 18 &= 0 \\
 (x+6)(x-3) &= 0 \\
 x &= -6, 3
 \end{aligned}$$

$$\begin{aligned}
 -(x^2 + 6x) &= 3x + 18 \\
 -x^2 - 6x &= 3x + 18 \\
 -x^2 - 9x - 18 &= 0 \\
 x^2 + 9x + 18 &= 0 \\
 (x+6)(x+3) &= 0 \\
 x &= -6, -3
 \end{aligned}$$

$$x = -6, 3, -3$$