

Complex Numbers

- Complex Numbers are based on a number whose square is -1.
- The **imaginary unit** i :

$$i = \sqrt{-1}$$

$$i^2 = -1$$
- Imaginary numbers and real numbers together make up the set of *complex numbers*.
- Complex numbers can be written in the form: $a + bi$
 - Where a and b are real numbers
- In *COMPLEX number plane*, to Graph (a, b) which represents $a + bi$:
 - Locate real part on horizontal axis and imaginary part on vertical axis.
- To *add* or *subtract* complex numbers, combine real parts together and combine imaginary parts together.
- Multiply* complex numbers/binomials the same as you do real numbers.
- $a + bi$ and $a - bi$ are called *complex conjugates* of each other. Their product is a real number! We often multiply the numerator and denominator by the conjugate of the denominator to get rid of the radical (i) in the denominator.

Examples

- How do you write $\sqrt{-24}$ using the imaginary unit i ?

$$i\sqrt{24} = i\sqrt{4 \cdot 6} = 2i\sqrt{6}$$
- What is each sum or difference?
 - $(2 + i) + (-3 + 3i) = 2 + i - 3 + 3i$
 $2 + (-3) + i + 3i$
 $-1 + 4i$
 - $(5 + i) + (5 - i)$
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 - $(-6 - 2i) - (4 + 2i)$
 $-6 - 2i - 4 - 2i = -10 - 4i$
- What is each product?
 - $-i(4 - 8i)$
 $-4i + 8i^2 = \boxed{-4i - 8}$
 - $(5 - 7i)(-4 + 3i)$
 $-20 - 15i + 28i + 21i^2$
 $-20 + 13i - 21$
 $\boxed{-41 + 13i}$

- What is each quotient?

$$\circ \frac{7i}{(8+i)} \cdot \frac{(8-i)}{(8-i)} = \frac{56i - 7i^2}{64 - i^2} = \frac{56i + 7}{64 + 1} = \boxed{\frac{56i + 7}{65}}$$

$$\circ \frac{(5+2i)(3+2i)}{(3-2i)(3+2i)} = \frac{15+10i+6i+4i^2}{9-4i^2}$$

$$= \frac{15+16i-4}{9+4} = \boxed{\frac{11+16i}{13}}$$

- What are the solutions of $3x^2 + 12 = 0$?

$$\frac{3(x^2+4)}{3} = \frac{0}{3}$$

$$x^2 + 4 = 0$$

$$-4 \quad -4$$

$$\sqrt{x^2} = \sqrt{-4}$$

$$x = \pm i\sqrt{4} = \boxed{\pm 2i}$$

- What are the solutions of $-x^2 + 4x - 5 = 0$?

$$\frac{1(x^2 - 4x + 5)}{-1} = \frac{0}{-1}$$

$$x^2 - 4x + 5 = 0$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{-4}}{2} = \frac{4 \pm 2i}{2}$$

$$x = \frac{4 \pm 2i}{2} = \frac{2(2 \pm i)}{2} = \boxed{2 \pm i}$$

$$\frac{3\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{6}}{\sqrt{2^2}} = \boxed{\frac{3\sqrt{6}}{2}}$$

$$\frac{2\sqrt[3]{3}}{\sqrt[3]{23}} \cdot \frac{\sqrt[3]{23^2}}{\sqrt[3]{23^2}} = \frac{2\sqrt[3]{587}}{\sqrt[3]{23^3}} = \frac{2\sqrt[3]{587}}{23}$$

$$\frac{2}{(1-\sqrt{3})} \cdot \frac{(1+\sqrt{3})}{(1+\sqrt{3})} = \frac{2(1+\sqrt{3})}{1-9} = \frac{2(1+\sqrt{3})}{1-3} = \frac{2(1+\sqrt{3})}{-2} = \frac{-(1+\sqrt{3})}{1}$$

Multiply By the Conjugate
Rationalizing the Denominator

$$\frac{7}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{7\sqrt{7}}{7} = \sqrt{7}$$

$$\frac{1}{\sqrt{8}} = \frac{1}{\sqrt{4 \cdot 2}} = \frac{1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2(2)} = \frac{\sqrt{2}}{4}$$

Ex: $x^2 + 10x + 40 = 0$ Solve by Completing the Square

$$x^2 + 10x + 5^2 - 5^2 + 40 = 0$$

$$(x+5)^2 + 15 = 0$$

$\quad -15 \quad -15$

$$\sqrt{(x+5)^2} = \sqrt{-15}$$

$$x+5 = \pm i\sqrt{15}$$

$$\boxed{x = -5 \pm i\sqrt{15}}$$