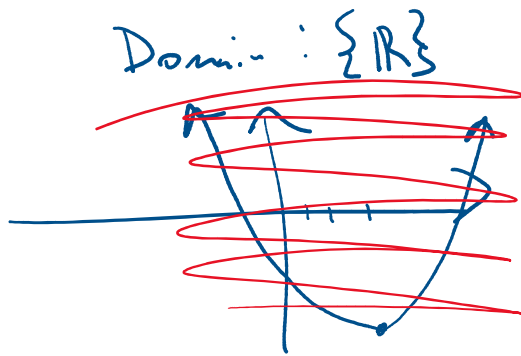


Exploring Quadratic Functions & Their Properties

- **Standard Form of a Polynomial Function**
 - $P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$
 - Notice: State in Decreasing order of power
- **Format of Quadratics/Parabola's**
 - Standard Form: $f(x) = ax^2 + bx + c$ *Solving*
 - Vertex Form: $f(x) = a(x-h)^2 + k$ *Graphing*
- **Axis of Symmetry:** A *LINE* that divides the parabola into 2 mirror images ($x = h$).
- **Vertex:** *Point* where parabola hits axis of symmetry (h, k).
- **Parent quadratic function:** $f(x) = x^2$
- **Transformations of the parent quadratic**
 - Increasing a , narrows graph; Decreasing a , widens graph
 - a positive = up; a negative = down
 - h shifts graph left and right
 - k shifts graph up and down
- For $y = \frac{1}{2}(x-3)^2 - 5$, what are the vertex, axis of symmetry, minimum or maximum value, domain, and range? *Vertex (3, -5) AOS: $x=3$ minimum at (3, -5)*



Range: $\{R \mid y \geq -5\}$

- **Information from Standard Form of Quadratic**
 - Axis of symmetry: $x = \frac{-b}{2a}$
 - **Vertex:** $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$
- What are the vertex, axis of symmetry, maximum or minimum value, and range of $y = -x^2 + 6x + 3$?
 - $a = -1$ $b = 6$ $c = 3$*
 - $x = \frac{-6}{2(-1)} = 3$*
 - $y = -(3)^2 + 6(3) + 3 = 12$*
 - Vertex (3, 12)*
 - maximum at (3, 12)*
 - AOS: $x = 3$*
 - Range $\{R \mid y \leq 12\}$*

- Sketch the graph of $f(x) = 3x^2 - 6x + 4$. Identify the vertex and the axis of symmetry.

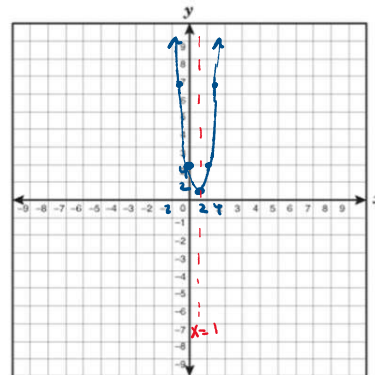
$$a = 3 \quad b = -6 \quad c = 4$$

$$\text{Vertex: } x = \frac{-(-6)}{2(3)} = 1 \quad ; \quad y = 3(1)^2 - 6(1) + 4 = 1$$

$$(1, 1)$$

$$\text{Aos: } x = 1$$

x	y
0	4
-1	$3(-1)^2 - 6(-1) + 4 = 13$



- Sketch the graph of $f(x) = -x^2 + 6x - 8$. Identify the vertex and x-intercepts.

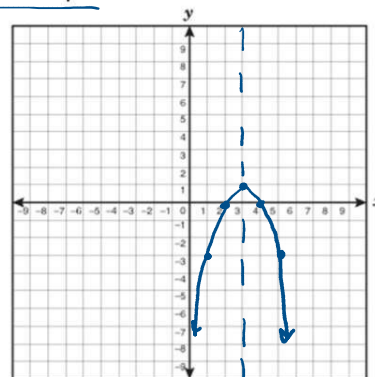
$$a = -1 \quad b = 6 \quad c = -8$$

$$x = \frac{-6}{2(-1)} = 3 \quad f(3) = y = -(3)^2 + 6(3) - 8 = -9 + 18 - 8 = 1$$

$$\text{Aos: } x = 3$$

$$\text{x-intercepts } (2, 0) \quad (4, 0)$$

x	y
1	$-(1)^2 + 6(1) - 8 = -3$
2	$-(2)^2 + 6(2) - 8 = 0$



- Write the standard form of the quadratic function whose graph is a parabola with vertex $(-4, 11)$ and passes through the point $(-6, 15)$.

$$y = a(x - h)^2 + k$$

$$y = a(x - (-4))^2 + 11$$

$$y = a(x + 4)^2 + 11 \quad \left\{ \begin{array}{l} y \\ a \\ x \end{array} \right. \checkmark$$

$$15 = a(-6 + 4)^2 + 11$$

$$15 = a(4) + 11$$

$$-11 \quad -11$$

$$\frac{4}{4} = \frac{4a}{4}$$

$$a = 1$$

$$y = 1(x + 4)^2 + 11$$

$$y = (x + 4)(x + 4) + 11$$

$$y = x^2 + 4x + 4x + 16 + 11$$

$$y = x^2 + 8x + 27$$

- How to go from one format to the other

○ $f(x) = ax^2 + bx + c$ TO $f(x) = a(x - h)^2 + k$

▪ COMPLETE THE SQUARE!!!

○ $f(x) = a(x - h)^2 + k$ TO $f(x) = ax^2 + bx + c$

▪ SQUARE BINOMIAL, DISTRIBUTE a TERM, COMBINE LIKE TERMS

- Completing the Square

- Method:

- Set up in format of: $y = ax^2 + bx + c$ (Notice: **a must be 1**)
- Take half of the middle term (b)

- Square it (b^2)
- Add it and Subtract it
- Factor perfect trinomial

$$y = x$$

$$y = x + 3 - 3$$

$$y + 3 = y + 3$$

- The first 3 terms (trinomial) are now "perfect" square. Sign comes from original

"b" term

- Combine leftover terms to obtain your k value
- Remember, the a value is the same in both standard form and vertex form.

$$y = a(x-h)^2 + k$$

- Convert to Vertex form: $y = x^2 + 12x + 27$?

$$y = x^2 + 12x + 6^2 - 6^2 + 27$$

$$y = (x+6)^2 - 36 + 27$$

$$y = (x+6)^2 - 9$$

$(-6, -9)$

- What value completes the square for $x^2 + 14x$?

$$7^2 = 49$$

- Graph by Completing the Square to obtain Vertex form. $y = x^2 + 6x - 1$?

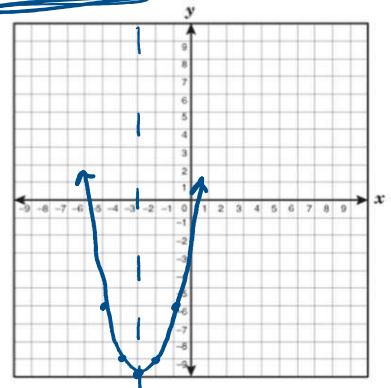
$$y = x^2 + 6x + 3^2 - 3^2 - 1$$

$$y = (x+3)^2 - 9 - 1$$

$$y = (x+3)^2 - 10$$

$(-3, -10)$

x	y
-2	-9
-1	-6



- What is $y = x^2 - 10x + 4$ in vertex form? Name the vertex and the y -intercept.

$$y = x^2 - 10x + 5^2 - 5^2 + 4$$

$$y = (x-5)^2 - 25 + 4$$

$$y = (x-5)^2 - 21$$

$$(5, -21)$$

$$y\text{-intercept } (0, 4)$$

- Convert $y = 2(x-3)^2 + 5$ to standard form of a quadratic.

$$y = 2(x-3)(x-3) + 5$$

$$y = 2[x^2 - 3x - 3x + 9] + 5$$

$$y = 2[x^2 - 6x + 9] + 5$$

$$y = 2x^2 - 12x + 18 + 5$$

$$y = 2x^2 - 12x + 23$$

- Convert $y = 2(x-3)^2 + 5$ to standard form of a quadratic.

$$y = 2(x-3)(x-3) + 5$$

$$y = 2[x^2 - 3x - 3x + 9] + 5$$

$$y = 2[x^2 - 6x + 9] + 5$$

$$\rightarrow y = 2x^2 - 12x + 18 + 5$$

$$y = 2x^2 - 12x + 23$$

$$y = 2x^2 - 12x + 17$$

$$y = 2(x^2 - 6x) + 17$$

$$y = 2(x^2 - 6x + 9 - 9) + 17$$

$$y = 2(x-3)^2 - 18 + 17$$

$$y = 2(x-3)^2 - 1$$