

Quadratic Equations and Applications

Methods: Factoring, Quadratic Equation, Completing the Square, and Graphing

- Factoring:

- Remember: First set equal to zero, *unless* no middle term
- See what you can "pull out" (ie, common factor)
- Setup parenthesis
- Setup signs (look at sign of third term in quadratic equation)
- Solve "F" (first term)
- Solve "L" (last term)
- Check "O" and "I" (outer and inner terms)
 - Alter as needed.
 - Right number-Wrong sign—just switch the signs
 - Wrong number-Try other factors of "L" term and test again
- Group like factors, if required (ie, $(x-1)(x-1) = (x-1)^2$)
- *Perfect Square Trinomials*: $a^2 + 2ab + b^2 = (a+b)^2$ AND $a^2 - 2ab + b^2 = (a-b)^2$
- *Difference of Perfect Squares*: $a^2 - b^2 = (a-b)(a+b)$
 - NOTICE: No middle term! EX: $4x^2 - 25 = 0$ $\frac{4x^2}{4} = \frac{25}{4}$ $\sqrt{x^2} = \sqrt{\frac{25}{4}}$ $x = \pm \frac{5}{2}$
- When specific form of quadratic (ie, difference of perfect squares), move constant to righthand side, isolate x^2 term, and take square root of both sides. Don't forget, +
- Hint: If $a = -1$, you can pull out -1 factor to enable easy factoring.
- Don't forget what solving a quadratic means. Represents graphically.

○ Solve: $x^2 + 7x + 6 = 0$

$$(x+6)(x+1) = 0$$

$\begin{array}{c} \swarrow \quad \searrow \\ +6x \quad +1x \\ \hline \end{array}$

$$\begin{array}{l} x+6=0 \quad x+1=0 \\ -6 \quad -1 \\ \hline x=-6 \quad x=-1 \end{array}$$

○ Solve: $4x^2 - 28x = 0$

$$4x(x-7) = 0$$

$$\begin{array}{l} \frac{4x}{4} = \frac{0}{4} \quad x-7=0 \\ \quad \quad \quad +7 \quad +7 \\ \hline x=0 \quad x=7 \end{array}$$

○ Solve: $2x^2 + 5x - 12 = 0$

$$(2x-3)(x+4) = 0$$

$\begin{array}{c} \swarrow \quad \searrow \\ -3x \quad +8x \\ \hline \end{array}$

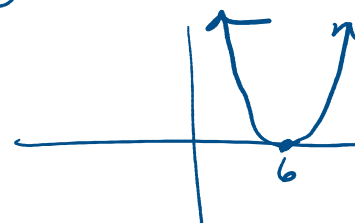
$$\begin{array}{l} 2x-3=0 \quad x+4=0 \\ +3 \quad +3 \quad -4 \quad -4 \\ \hline 2x = \frac{3}{2} \quad x = -4 \\ \hline x = \frac{3}{2} \quad x = -4 \end{array}$$

○ Solve: $x^2 - 12x + 36 = 0$

$$(x-6)(x-6) = 0$$

$$(x-6)^2$$

$$\begin{array}{l} x-6=0 \quad x-6=0 \\ +6 \quad +6 \\ \hline x=6 \end{array}$$



- Solve: $81x^2 - 100 = 0$ (2 Methods)

$$(9x + 10)(9x - 10) = 0$$

$$9x + 10 = 0 \quad 9x - 10 = 0$$

$$\begin{array}{r} -10 \\ 9x = -10 \\ \hline x = -\frac{10}{9} \end{array} \quad \begin{array}{r} +10 \\ 9x = \frac{10}{9} \\ \hline x = \frac{10}{9} \end{array}$$

$$\boxed{x = -\frac{10}{9}} \\ \boxed{x = \frac{10}{9}}$$

$$81x^2 - 100 = 0$$

$$+100 \quad +100$$

$$\frac{81x^2}{81} = \frac{100}{81}$$

$$\sqrt{x^2} = \sqrt{\frac{100}{81}} \quad x = \pm \sqrt{\frac{100}{81}} = \pm \frac{10}{9}$$

- Solve: a) $3x^2 + 5 = 20$

$$\begin{array}{r} -5 \\ 3x^2 = 15 \\ \hline x^2 = \frac{15}{3} \\ \hline x = \pm \sqrt{5} \end{array}$$

- b) $8x^2 - 3 = 29$

$$\begin{array}{r} +3 \quad +3 \\ 8x^2 = 32 \\ \hline x^2 = \frac{32}{8} \\ \hline x = \pm \sqrt{4} \\ \hline \boxed{x = \pm 2} \end{array}$$

- What are the *solutions* of the quadratic equation: $x^2 + 3x - 18 = 0$

$$(x + 6)(x - 3) = 0$$

$$\begin{array}{r} +6x \\ -3x \\ \hline \end{array}$$

$$x + 6 = 0 \quad x - 3 = 0$$

$$\boxed{x = -6 \quad x = 3}$$

- Solve: $10x^2 + 2x - 46 = x - 4$

$$\begin{array}{r} -x \quad +4 \quad -x \quad +4 \\ 10x^2 + x - 42 = 0 \\ (10x + 21)(x - 2) = 0 \end{array}$$

$$\begin{array}{r} +21x \\ -20x \\ \hline \end{array}$$

$$10x + 21 = 0 \quad x - 2 = 0$$

$$\begin{array}{r} -21 \\ 10x = -21 \\ \hline x = -\frac{21}{10} \end{array} \quad \boxed{x = 2}$$

- What are the zero's of: $5x^2 - 8 = 18x$

$$5x^2 - 18x - 8 = 0$$

$$(5x + 2)(x - 4) = 0$$

$$\begin{array}{r} +2x \\ -20x \\ \hline \end{array}$$

$$5x + 2 = 0 \quad x - 4 = 0$$

$$\begin{array}{r} -2 \quad -2 \\ 5x = -2 \\ \hline x = -\frac{2}{5} \end{array} \quad \begin{array}{r} +4 \quad +4 \\ x = 4 \end{array}$$

• Completing the Square

- Sometimes an equation shows a perfect square trinomial (remember—these are ones with no middle term!) equal to a constant. IF SO, factor the perfect square trinomial; Take square root of BOTH sides! Don't forget $\pm$; Solve.
- What if we are not given a perfect square trinomial??? We can *Complete the Square* and obtain one!
- Method:
 - Set up in format of: $x^2 + bx = c$ (Notice: a must be 1)
 - Take half of the middle term (b)
 - Square it (b^2)
 - Add it to both sides

▪ To then solve:

- Factor perfect trinomial on left side.
- Take square root of both sides (not forgetting $\pm$)
- Solve for x .

○ What is the solution of $x^2 + 12x + 36 = 9$?

$$\begin{array}{l}
 x^2 + 12x + 36 = 9 \\
 \sqrt{(x+6)^2} = \sqrt{9} \\
 x+6 = \pm 3 \\
 x = -6 \pm 3 \\
 \boxed{x = -6+3 \quad x = -6-3} \\
 \boxed{x = -3 \quad x = -9}
 \end{array}
 \quad
 \begin{array}{l}
 x^2 + 12x + 36 = 9 \\
 x^2 + 12x + 36 - 36 = 9 - 36 \\
 x^2 + 12x + 0 = -27 \\
 x^2 + 12x = -27 \\
 x^2 + 12x + 36 = -27 + 36 \\
 (x+6)^2 = 9 \\
 \sqrt{(x+6)^2} = \sqrt{9} \\
 x+6 = \pm 3 \\
 \boxed{x = -3, -9}
 \end{array}$$

○ Solve $3x^2 + 18x - 3 = 0$ by completing the square.

$$\begin{array}{l}
 3x^2 + 18x - 3 = 0 \\
 3(x^2 + 6x + 3^2 - 3^2) - 3 = 0 \\
 3(x+3)^2 - 27 - 3 = 0 \\
 3(x+3)^2 = 30 \\
 \frac{3(x+3)^2}{3} = \frac{30}{3} \\
 \sqrt{(x+3)^2} = \sqrt{10} \\
 x+3 = \pm \sqrt{10} \\
 \boxed{x = -3 \pm \sqrt{10}}
 \end{array}$$

• Quadratic Formula

- $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- Discriminant ($b^2 - 4ac$) tells us about solutions we will receive.
 - $b^2 - 4ac > 0$ tells us 2 real solutions
 - $b^2 - 4ac = 0$ tells us 1 real solution
 - $b^2 - 4ac < 0$ tells us no real solutions (2 imaginary solutions)
- EX: Determine the solutions using the quadratic formula. $5x^2 - 2x = 2$

$$5x^2 - 2x - 2 = 0$$

$$\begin{array}{l}
 a = 5 \\
 b = -2 \\
 c = -2
 \end{array}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(5)(-2)}}{2(5)}$$

$$x = \frac{2 \pm \sqrt{4+40}}{10}$$

$$\begin{aligned}
 x &= \frac{2 \pm \sqrt{44}}{10} = \frac{2 \pm \sqrt{4 \cdot 11}}{10} = \frac{2 \pm \sqrt{2^2 \cdot 11}}{10} = \frac{2 \pm 2\sqrt{11}}{10} \\
 &= \frac{2(1 \pm \sqrt{11})}{10} = \frac{1 \pm \sqrt{11}}{5}
 \end{aligned}$$

- EX: You sell wrapping paper as a charity fundraiser. The equation $p = -6x^2 + 280x - 1200$ models the total profit p as a function of the price x per roll of paper. What is the smallest amount in dollars you can charge per roll of wrapping paper to make a profit of \$1500.

$$-6x^2 + 280x - 1200 \stackrel{?}{=} 1500$$

$$-6x^2 + 280x - 2700 = 0$$

★ Use Calculator

- EX: What is the number of real solutions of $-x^2 + 14x = 49$?

$$-x^2 + 14x - 49 = 0 \quad a = -1 \quad b = 14 \quad c = -49$$

Discriminant: $b^2 - 4ac$

$$(14)^2 - 4(-1)(-49) = 196 - 196 = 0$$

1 real solution

- Common Application of quadratics – an object falling or being projected into the air. General position function is: $s = -16t^2 + v_0t + s_0$.

- EX: A rocket is launched from the ground with an initial vertical velocity of 150 ft/s. The function $h = -16t^2 + 150t$ models the height in feet of the rocket at time t in seconds. Will the rocket reach a height of 300 ft? Explain your answer.

Calculator → RA setup here.

- EX: From 2009 through 2015, the number of monthly active Facebook users F (in millions) worldwide can be approximated by the quadratic equation $F = -10.024t^2 + 448.79t - 2990.5$, $9 \leq t \leq 15$ where t represents the year, with $t = 9$ corresponding to 2009. According to the model, in which year did the number of users reach one billion (1000 million)?

On calculator Examples