

Quadratic Equations and Applications

Methods: Factoring, Quadratic Equation, Completing the Square, and Graphing

- Factoring:
 - Remember: First set equal to zero, *unless* no middle term
 - See what you can “pull out” (ie, common factor)
 - Setup parenthesis
 - Setup signs (look at sign of third term in quadratic equation)
 - Solve “F” (first term)
 - Solve “L” (last term)
 - Check “O” and “I” (outer and inner terms)
 - Alter as needed.
 - Right number-Wrong sign—just switch the signs
 - Wrong number-Try other factors of “L” term and test again
 - Group like factors, if required (ie, $(x - 1)(x - 1) = (x - 1)^2$)
 - *Perfect Square Trinomials*: $a^2 + 2ab + b^2 = (a + b)^2$ AND $a^2 - 2ab + b^2 = (a - b)^2$
 - *Difference of Perfect Squares*: $a^2 - b^2 = (a - b)(a + b)$
 - NOTICE: No middle term! EX: $4x^2 - 25$
 - When specific form of quadratic (ie, difference of perfect squares), move constant to righthand side, isolate x^2 term, and take square root of both sides. Don’t forget, $\pm$
 - Hint: If $a = -1$, you can pull out -1 factor to enable easy factoring.
 - Don’t forget what solving a quadratic means. Represents graphically.
- Solve: $x^2 + 7x + 6 = 0$
- Solve: $4x^2 - 28x = 0$
- Solve: $2x^2 + 5x - 12 = 0$
- Solve: $x^2 - 12x + 36 = 0$

○ Solve: $81x^2 - 100 = 0$ (2 Methods)

○ Solve: a) $3x^2 + 5 = 20$

b) $8x^2 - 3 = 29$

○ What are the *solutions* of the quadratic equation: $x^2 + 3x - 18 = 0$

○ Solve: $10x^2 + 2x - 46 = x - 4$

○ What are the zero's of: $5x^2 - 8 = 18x$

- *Completing the Square*

- *Sometimes* an equation shows a perfect square trinomial (remember—these are ones with no middle term!) equal to a constant. IF SO, factor the perfect square trinomial; Take square root of BOTH sides! Don't forget $\pm$; Solve.
- What if we are not given a perfect square trinomial??? We can *Complete the Square* and obtain one!
- Method:
 - Set up in format of: $x^2 + bx = c$ (Notice: a must be 1)
 - Take half of the middle term (b)
 - Square it (b^2)
 - Add it to both sides

- To then solve:
 - Factor perfect trinomial on left side.
 - Take square root of both sides (not forgetting $\pm$)
 - Solve for x .
- What is the solution of $x^2 + 12x + 36 = 9$?

- Solve $3x^2 + 18x - 3 = 0$ by completing the square.

- *Quadratic Formula*

- $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- Discriminant ($b^2 - 4ac$) tells us about solutions we will receive.
 - $b^2 - 4ac > 0$ tells us 2 real solutions
 - $b^2 - 4ac = 0$ tells us 1 real solution
 - $b^2 - 4ac < 0$ tells us no real solutions (2 imaginary solutions)
- EX: Determine the solutions using the quadratic formula. $5x^2 - 2x = 2$

- EX: You sell wrapping paper as a charity fundraiser. The equation $p = -6x^2 + 280x - 1200$ models the total profit p as a function of the price x per roll of paper. What is the smallest amount in dollars you can charge per roll of wrapping paper to make a profit of \$1500.

- EX: What is the number of real solutions of $-x^2 + 14x = 49$?

- Common Application of quadratics – an object falling or being projected into the air. General position function is: $s = -16t^2 + v_0t + s_0$.
 - EX: A rocket is launched from the ground with an initial vertical velocity of 150 ft/s. The function $h = -16t^2 + 150t$ models the height in feet of the rocket at time t in seconds. Will the rocket reach a height of 300 ft? Explain your answer.

 - EX: From 2009 through 2015, the number of monthly active Facebook users F (in millions) worldwide can be approximated by the quadratic equation $F = -10.024t^2 + 448.79t - 2990.5$, $9 \leq t \leq 15$ where t represents the year, with $t = 9$ corresponding to 2009. According to the model, in which year did the number of users reach one billion (1000 million)?